JA-003-1016001 Seat No. ________

B. Sc. (Sem. VI) (CBCS) (W.E.F.) Examination

August – 2019

Mathematics : Paper - VIII

(Graph Theory & Complex Analysis - II) (New Course)

Faculty Code : 003

Subject Code : 1016001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Figures on the right side indicate marks.

1 (a) Answer the following questions briefly : **4**

(1) Every complete graph is regular graph but converse is not true illustrate with an example.

(2) Define : Forest and Minimal connected graph.

(3) What is the the number of pendent vertices in any binary tree with n vertices?

(4) Prove that $e \le 3n - 6$ is only necessary but not sufficient.

(b) Attempt any one out of two : **2**

(1) Find the number of vertices of 4-regular graph and the number of edges 10 and sketch it.

(2) (a) Define : Planner graph with an example.

(b) Draw the Kuratowski two graph.

(c) Attempt any one out of two : **3**

(1) Let G be a (n, m) graph. $\delta = \min\left\{d(v_i)/v_i \in V\right\}$ and $\Delta = \max\left\{d(v_i)/v_i \in V\right\}$ then prove that $\delta \le \frac{2m}{n} \le \Delta$.

(2) Prove that the number of edges in tree with n-vertices is (n-1).

(d) Attempt any one out of two : **5**

(1) Explain Konigsberg bridge problem and the solution give by Euler.

(2) (a) Define Length of path and prove that every path is open walk but converse is not true.

(b) G is tree if and only if there is one path between every pair of vertices. Prove this.

2 (a) Answer the following questions briefly : **4**

(1) Give an example of 3- chromatic, 2- chromatic, 1- chromatic graph.

(2) Define: Acyclic diagraph and give its example.

(3) Define: Minimal covering of G.

(4) Define: Basis of W_G and w_s.

(b) Attempt any one out of two : **2**

(1) Define: Path matrix and give its example.

(2) Prove that $(W_S, \oplus)$ is commutative group.

(c) Attempt any one out of two : **3**

(1) (a) Kuratowski's Second graph $K_{3,3}$ has _______ edges. Fill in the blank.

(b) A simple connected graph with 4 vertices can have atmos _________ edges. Fill in the blank.

(c) Find the Regions(faces) of a connected planar graph with 4 vertices and 6 edges.

(2) Explain dual graph for planner graph and give any two properties of it.

(d) Attempt any one out of two : **5**

(1) If G is a graph with n-vertices, e- edges , f-faces and K- components, then show that $n - e + f = K + 1$.

(2) Define cut-set vector and in usual notation prove that $\left(W_s, \oplus, '\right)$ is a subspace of W_G over field $GF_2 = \{0, 1\}$.

3 (a) Answer the following questions briefly : **4**

(1) Find the region of convergence and radius of convergence of the series $\Sigma_{n=1}^{\infty} \dfrac{z^n}{3^n + 1}$.

(2) Define : Complex series.

(3) Define : Power series.

(4) Expand $\dfrac{1}{1+z}$ in Maclaurian's series.

(b) Attempt any one out of two : **2**

(1) Expand $e^z z^{-1} \left(1 + z^2\right)^{-1}$ in Laurentz's series for $|z| < 1$.

(2) Prove that $\dfrac{1}{z^2} = \dfrac{1}{4} + \dfrac{1}{4} \Sigma_{n=1}^{\infty} (-1)^n (n+1) \left(\dfrac{z-2}{2}\right)^n$.

(c) Attempt any one out of two : **3**

(1) State and prove Laurentz's infinity series for an analytic function.

(2) State and prove necessary and sufficient condition for complex sequence $\{Z_n\}$ to be convergent.

(d) Attempt any one out of two : **5**

(1) In usual notation prove that cosh $(Z + z^{-1})$ = $a_0 + \sum_{n=1}^{\infty} a_n \left(Z^n + Z^{-n}\right)$.

(2) Sate the Taylor's theorem and expand $\frac{2z^3+1}{z^3+z}$ in Taylor series for $Z_0 = i$.

4 (a) Answer the following questions briefly **4**

(1) Define : Isolated singular point.

(2) Define : Zero of complex function.

(3) Define : Residue of f (z) at pole z_0.

(4) Evaluate : $\int_c \frac{5z-2}{z(z-1)}\, dz,\ C : |z| = 2.$

(b) Attempt any one out of two : **2**

(1) Define: Pole of complex function.

(2) Prove that $\int_0^{\infty} \frac{\cos ax}{x^2+1} = \frac{\pi}{2}\, e^{-4}.$

(c) Attempt any one out of two : **3**

(1) Using residue theorem, find the value of $\int_{-\infty}^{\infty} \frac{dz}{\left(1+x^2\right)^3}$.

(2) State and prove Cauchy's Residue theorem.

(d) Attempt any one out of two : **5**

(1) Expand $f(z) = \frac{1}{z^2 \sin h\, z}$ in laurent's series for $z_0 = 0$ and hence deduce that

$$\int_c \frac{1}{\sinh z}\, dz = 2\pi i \ \text{ and } \int_c \frac{1}{z^2 \sin h\, z}\, dz = -\frac{\pi}{3} i.$$

(2) If Z_0 is the m^{th} order pole of complex function $f(z)$ then prove that

$$\operatorname{Res}\left(f(z), z_0\right) = \frac{\phi^{m-1}\left(z_0\right)}{(m-1)!}, \text{ where } \phi(z) = \left(z - z_0\right)^m f(z).$$

5 (a) Answer the following questions briefly : **4**

(1) Define : Singular point.

(2) If the series ΣZ_n is absolute convergent then ΣZ_n is also convergent.

(3) Expand e^z in increasing power of z.

(4) Define : Region of Convergence series.

(b) Attempt any one out of two : **2**

(1) Expand $\dfrac{1}{(z-1)(z-2)}$ in laurent's series for $1 < |z| < 2$.

(2) Find $\operatorname{Res}\left(\dfrac{e^z}{z(z+1)}, 0\right)$.

(c) Attempt any one out of two : **3**

(1) Prove that $\displaystyle\int_0^{2\pi} \frac{d\theta}{(2+\cos\theta)^2} = \frac{4\pi}{3\sqrt{3}}$.

(2) Prove that $\displaystyle\int_0^{\infty} \frac{x \sin ax}{x^2 + k^2}\, dx = \frac{\pi}{2} e^{-ak}; a, k \geq 0$.

(d) Attempt any one out of two : **5**

(1) Prove that $\displaystyle\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta} = \frac{2\pi}{\sqrt{1-a^2}}; -1 < a < 1$.

(2) By Residue theorem find the value of

$$\int_0^{\pi} \frac{d\theta}{(a + \cos\theta)^2}; a > 1.$$